Growth, Uncertainty and Business Cycles in an Overlapping Generations Economy

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Overview

Great recession saw a large, persistent fall in GDP; brief, small decline in TFP.

- Large responses in GDP and investment from credit shocks (Khan and Thomas 2013, Jo 2021 and Khan, Thomas and Senga 2017).
- Misallocation rises across a distribution of firms, given borrowing constraints, and the return on savings falls.

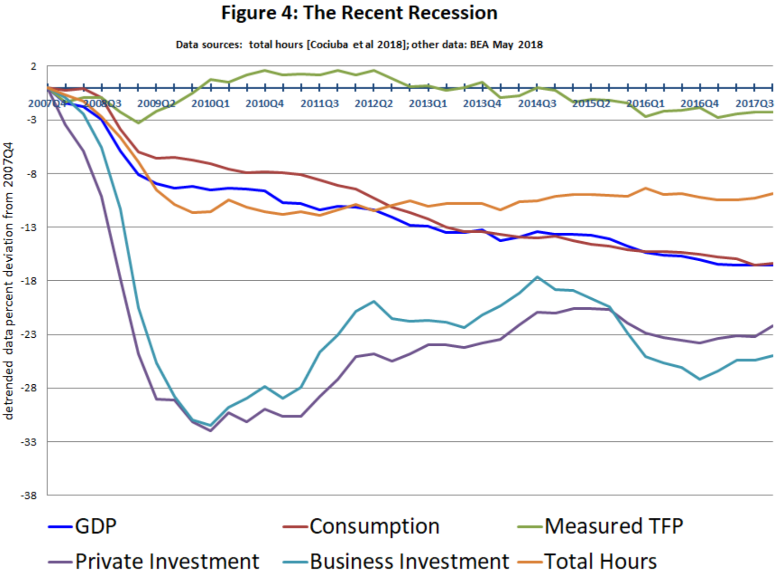
Consumption difficult to explain in complete markets models.

Persistent lack of recovery despite TFP growth.

Explore the role of *growth shocks* and an increase in uninsurable risk.

Introduce a strong precautionary savings motive into an incomplete markets economy.

2007 US Recession



Approach

- A Quantitative Overlapping Generations Economy.
- Leptokurtic distribution of earnings shocks.
- Unemployment risk rises in recessions.
- Growth shocks to TFP, Uncertainty Shocks to Labour Force Participation
- time-varying price of capital, non-separable utility

Explore aggregate consumption dynamics in this setting.

▸ literature

Earnings and Employment

Individuals of age j survive with probability ω_j, $j = 1, \ldots, J$.

Type $b = 1, \ldots, N(\beta)$ has subjective discount factor, β_b.

Employed $e \in \left\{e_1, \ldots, e_{N(e)}\right\}$ fraction of period with

$$\Pr(e = e_i) = \pi_i^e(z, u).$$

Labour productivity $\zeta \in \left\{\zeta_1, \ldots, \zeta_{N(\zeta)}\right\}$

$$\Pr\left\{\zeta' = \zeta_l, \zeta = \zeta_k\right\} = \pi_{kl} \geq 0.$$

Individual state is (j, b, S, e, ζ) with S holdings of aggregate capital.

Unemployment/ labour force risk varies with TFP growth, z, and uncertainty, u.

Working Households

Working age individuals, $j = 1, \ldots, J_r - 1$ solve

$$V^b\left(j, S, e_h, \zeta_k, A_{-1}, z_g, u_m, \mu\right) = \max_{C,S'}\Big(\left(1-\beta_b\right) C^{1-\sigma} \tag{1}$$

$$+\beta_b \omega_j \Big(\sum_{n=1}^{N(u)} \pi^u_{mn} \sum_{h=1}^{N(z)} \pi^z_{gh} \sum_{i=1}^{N(e)} \pi^e_i\left(z_g, u_m\right) \times$$

$$\sum_{l=1}^{N(\zeta)} \pi_{kl} V^b\left(j+1, S', e_i, \zeta_l, A, z_h, u_n, \mu'\right)^{1-\gamma}\Big)^{\frac{1-\sigma}{1-\gamma}}\Big)^{\frac{1}{1-\sigma}} \tag{2}$$

subject to

$$C + PS' \leq M_0\left(j, S, e_h, \zeta_k; A_{-1}, z_i, u_m, \mu\right),$$

FE involves the *certainty equivalent* of future value, with γ the coefficient of relative risk aversion.

retired household problem

Calibration earnings shocks

moment	data	continuous	discrete
std 1 year change	0.51	0.57	0.45
std 5 year change	0.78	0.79	0.72
skewness 1 year change	-1.07	0.00	-0.96
skewness 5 year change	-1.25	0.00	-0.11
kurtosis 1 year change	14.93	15.26	15.02
kurtosis 5 year change	9.51	8.42	5.55
fraction 1 year change $< 5\%$	0.306	0.28	0.42
fraction 1 year change $< 10\%$	0.488	0.60	0.56
fraction 1 year change $< 20\%$	0.665	0.83	0.84

Guvenen, Karahan, Ozkan and Song (2019)

- The 9 moments above are used to estimate a continuous stochastic process with innovations that have Poisson arrival rates.
- Next the support of a Markov Chain is estimated by simulating the continuous process.

Constructing a Markov Chain for income shocks

- $\lambda_i = 1$ probability p_i and $\varepsilon_i \sim \mathbf{N}(0, \sigma_i^2)$, Kaplan, Moll and Violante (2017)

$$
\begin{aligned}
\log \zeta &= \log z_1 + \log z_2 \\
\log z_i' &= \rho_i \log z_i + \lambda_i \varepsilon_i, i = 1, 2.
\end{aligned}
$$

ρ_1	ρ_2	p_1	p_2	σ_1	σ_2
-0.05	0.97	0.016	0.695	2.61	0.22

- Next, setting $N(\zeta) = 15$, estimate the support of a Markov chain to reproduce the above moments for the continuous process.
- Any support $\left\{\zeta_1, \ldots, \zeta_{N(\zeta)}\right\}$ used to sort a simulation of the continuous process implies a transition matrix.
- Simulating this Markov chain yields the moments for the discretised stochastic process for income shocks.

Calibration

unemployment rate is 5% on average, rises to 9% in a large recession

$N(e) = 3$ with mean, median duration of unemployment set to average of
15.9 weeks and 7.8 weeks (12.81 - 12.07)
31.6 weeks and 16 weeks (1.08 - 4.16).

$\pi_1^e(z)$ varies between 15.9 weeks in expansion and 31.6 weeks in recession

Shock to Labour Force Participation and Investment
$N(u) = 2$ with $\pi_{11}^u = 0.98$ and $\pi_{22}^u = 0.9$.

Target 3.44 percent fall in labour force participation (2007Q3 - 2015Q3). ▸ LFP

2% probability of labour force shock when $u = 2$ (return with probability 0.5).

Investment TFP falls 20 percent

Distribution of wealth

Table 3: Wealth distribution

	1	5	10	50	90	Gini	≤ 0
SCF	0.29	0.51	0.64	0.97	1.0	0.77	0.09
model	0.14	0.48	0.63	0.95	1.0	0.73	0.19

1. Reproduces concentration in top 5th percentile.
2. Insufficient concentration in right tail concentration, left tail too large.
3. High precautionary savings ($r = 5.8\%$ rises to 7.2% without income risk, 84% net worth ≤ 0)

▸ crra distribution

Backwards Induction Applied to QOLG economies

Aggregate state is a high-dimensional object, containing the distribution of households over age, employment, labour productivity and wealth.

Aggregate state space approximation *with simple forecasting rules* is difficult. Shocks redistribute wealth across different age groups and approximate aggregation doesn't hold.

I use state space approximation but allow for a non-parametric law of motion.

Stochastic Equilibrium solved using generalistion of Reiter (2010) feasible for OLG economies, see Kim (forthcoming).

At each point on approximate aggregate state grid, (z_g, u, m) given expectation $\widehat{m}'$ households solve decision rules $g(j, a, e_h, \zeta_k, z_g, u, m)$.

Backwards induction imposes consistency between household decision rules, given a proxy distribution $\mu(j, a, e, \zeta; z, u, m)$ at each (z, u, m), and m'.

Business cycles with TFP Growth Shocks

familiar cyclical behaviour

Table 4: Business Cycle Moments

	Z	Y	C	I	N	K	w
std	1.86	2.53	1.97	2.91	1.68	2.67	1.71
correlation	0.82	1.00	0.94	0.98	0.74	0.62	0.75

Results from 1000 period simulation, HP-filtered with weight 100

TFP Growth and Labour Force Uncertainty Shocks

	Z	Y	C	I	N	K	w
std	1.9	2.593	2.04	3.00	1.75	2.58	1.77
correlation	0.80	1.00	0.93	0.96	0.74	0.72	0.74

Results from 2000 period simulation, HP-filtered with weight 100

Shocks to TFP growth change employment risk but not labour force participation.
Adding labour force shocks (lower table), which are rare, has small effect.

business cycles comparison

Peak-to-trough: US 2007 recession and model

persistent shock to TFP growth

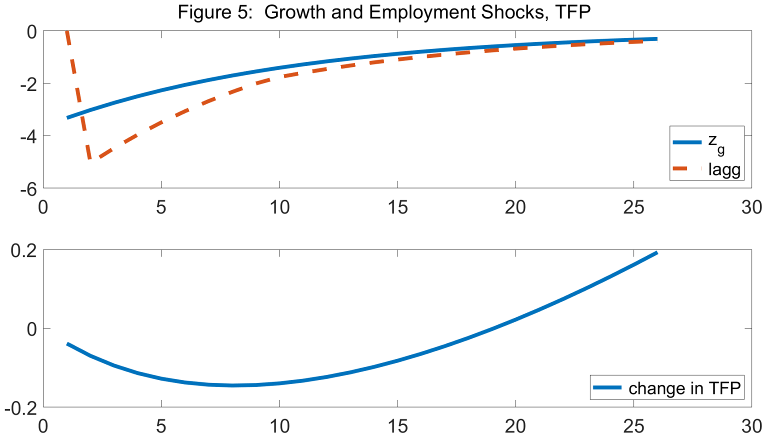
	trough	GDP	I	N	C	TFP
data	2009Q2	8.65	25.92	7.00	6.71	3.61
model		7.29	10.2	5.04	4.12	3.3

Growth shock causes a large fall in consumption

- 84 percent of the observed GDP drop, 61 percent of the fall in consumption using *empirically consistent shock to TFP growth*
- absence of misallocation and strong precautionary savings motive implies smaller investment response (combine with credit shocks)
- individual unemployment shocks reduce aggregate employment

TFP Growth and Employment Shocks

Nonmonotone response in TFP.

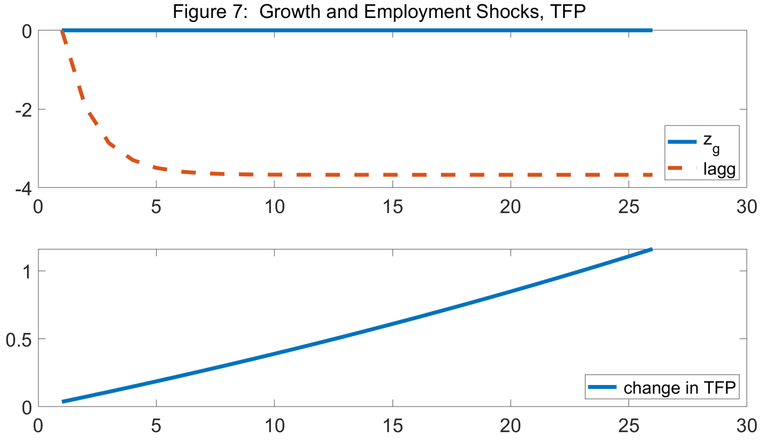


Unemployment risk changes with TFP growth.

Labour Force Participation Shock

persistent movements out of labour force and investment shocks

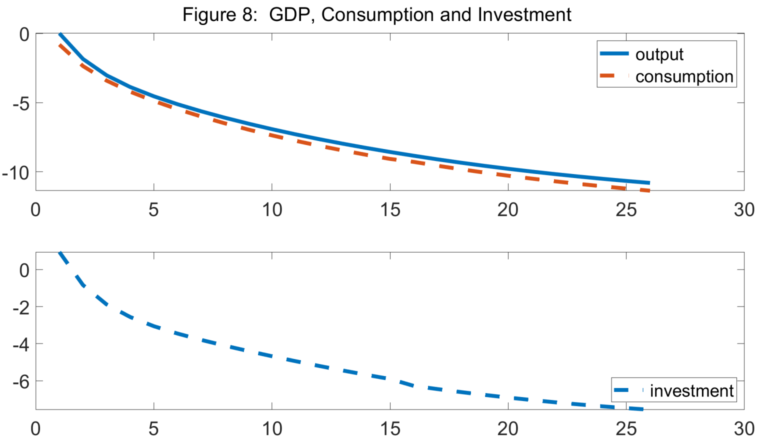
Lack of Recovery cannot be from trend growth shock



known a period before, no shock to TFP growth, 20 percent drop in inv. TFP

Labour Force Participation Shock

large, persistent declines in Consumption and GDP



precautionary savings dampens investment fall ▸ Uncertainty without Investment , ▸ Uncertainty with crra

Closing remarks

Business cycles in an QOLG with uninsurable employment risk.

Finite lifetimes imply large changes in individual consumption from income shocks.

Growth shocks imply persistent, nonmonotone movements in aggregate TFP.

Large monotone response in aggregate consumption; real rate is hump-shaped.

A rise in labour market uncertainty leads to a sharp fall in aggregate consumption; GDP fall requires additional investment shock.

Income and Wealth Inequality over Large Recessions

Krueger, Mitman and Perri (2017): inequality amplifies fall in consumption

Kim (forthcoming): risk of severe economic downturn increases precautionary savings.

Glover, Heathcote, Krueger and Rios-Rull (2017): large welfare costs upon old

Guerrieri and Lorenzoni (2015): credit shocks to borrowing limits

Stochastic equilibrium in Quantitative OLG models

Kubler and Krueger (2010)

Reiter (2010): Backwards Induction

Kim(forthcoming): Projection to QOLG models

Savings rates vary by age, older households are wealthier.

▸ approach

Sources of risk and inequality

- Incomplete markets and wealth inequality.

  Huggett (1993), Aiyagari (1994)

- Household differences in discounting

  Krusell and Smith (1998): infinite horizon

  Krueger, Mitman and Perri (2017): OLG with stochastic aging

  Hubmer, Krusell and Smith (2018): infinite horizon

- Cyclical household unemployment risk

  Krusell and Smith (1998)

  Krueger, Mitman and Perri (2017)

  Hubmer, Krusell and Smith (2018)

approach

Working-age and retired households

Individual $j = 1, \ldots, J_r - 1$, has worked for $j - 1$ periods,

$$\begin{aligned} C + PS' &\leq (P + D) S + (1 - \tau) W \zeta e + (1 - e) B \zeta \\ &\equiv M_0 (j, S, e, \zeta), \end{aligned}$$

Borrowing limit

$$S' \geq \underline{S},$$

Retired $j = J_r, \ldots, J$,

$$C + PS' \leq (P + D) S + B \equiv M_1 (j, S),$$

Pensions are lump sum, no annuities markets.

Aggregate State

TFP is $A^{1-\alpha} = (A_{-1}z)^{1-\alpha}$ with z a shock to TFP growth.

$\left\{ z_i, \left\{ \pi^z_{ij} \right\}_{j=1}^{N(z)} \right\}_{i=1}^{N(z)}$ is a Markov Chain.

u is the aggregate uncertainty state, $N(u) = 2$ and

$$\Pr\left\{ u' = u_j \mid u = u_i \right\} = \pi^u_{ij}$$

The aggregate state is (A_{-1}, z_i, u, μ) with μ a distribution over (j, b, S, e, ζ).

Individuals choose consumption and asset holdings given

$$\mu' = \Gamma\left(A_{-1}, z_i, u_m, \mu\right).$$

Retired Households

An individual of age J consumes cash on hand.

Retired people that began working age $j = J_r, \ldots, J-1$ periods ago solve

$$V^b\left(j, S, e_h, \zeta_k, A_{-1}, z_g, u_m, \mu\right) = \max_{C,S'}\Big(\left(1-\beta_b\right)C^{1-\sigma} \tag{3}$$

$$+\beta_b\omega_j\Big(\sum_{n=1}^{N(u)}\pi_{mn}^u\sum_{h=1}^{N(z)}\pi_{gh}^z\sum_{i=1}^{N(e)}\pi_i^e\left(z_g, u_m\right)\times$$

$$\sum_{l=1}^{N(\zeta)}\pi_{kl}V^b\left(j+1, S', e_i, \zeta_l, A, z_h, u_n, \mu'\right)^{1-\gamma}\Big)^{\frac{1-\sigma}{1-\gamma}}\Big)^{\frac{1}{1-\sigma}} \tag{4}$$

subject to

$$(5)$$

$$C + PS' \leq M_1\left(j, S; A_{-1}, z_i, u_m, \mu\right).$$

▸ working household problem

Investment Firm

diminishing marginal productivity of investment spending

$$J\left(K, A_{-1}, z_g, u_m, \mu\right) = \max_{K'}\Bigg(\left(r_0^k + 1 - \delta\right) K - \Big(P + D\Big) K \tag{6}$$
$$+PK' - \psi\left(K', K\right)$$
$$+ \sum_{n=1}^{N(z)} \pi_{mn}^u \sum_{h=1}^{N(z)} \pi_{gh}^z q\left(z_h, A_{-1}, z_g, u_m, \mu\right) J\left(K', A, z_h, u_n, \mu'\right)\Bigg)$$

Competitive investment firm

$$\begin{aligned} P\left(A_{-1}, z_g, u_m, \mu\right) &= \psi_1\left(K', K\right) \\ D\left(A_{-1}, z_g, u_m, \mu\right) &= r_0^k\left(A_{-1}, z_g, u_m, \mu\right) + 1 - \delta \\ &\quad - P\left(A_{-1}, z_g, u_m, \mu\right) - \psi_2\left(K', K\right). \end{aligned}$$

A time-varying price of capital without solving for $q\left(z_h, A_{-1}, z_g, u_m, \mu\right)$ (households vary in their marginal rates of intertemporal substitution).

Production and wages

Final goods firm rents capital and labour

$$\max_{K,N}\left(\left(A_{-1}z_g\right)^{1-\alpha}K^{\alpha}L^{1-\alpha}-r_0^k\left(A_{-1},z_g,u_m,\mu\right)K-W_0\left(A_{-1},z_g,u_m,\mu\right)L\right).$$

$$r_0^k\left(A_{-1},z_g,u_m,\mu\right) = \alpha\left(A_{-1}z_g\right)^{1-\alpha}K^{\alpha-1}L^{1-\alpha}, \quad (7)$$

$$W_0\left(A_{-1},z_g,u_m,\mu\right) = (1-\alpha)\left(A_{-1}z_g\right)^{1-\alpha}K^{\alpha}L^{-\alpha}. \quad (8)$$

Given employment shocks $\pi_i^e(z,u)$, aggregate employment L is determined by last period's z and u.

▸ working household problem

Calibration

unemployment rate is 5% on average, rises to 9% in a large recession

$N(e) = 3$ with mean, median duration of unemployment set to average of
15.9 weeks and 7.8 weeks (12.81 - 12.07)
31.6 weeks and 16 weeks (1.08 - 4.16).

$\pi_1^e(z)$ varies between 15.9 weeks in expansion and 31.6 weeks in recession

capital's share and depreciation: $\alpha = 0.36$ and $\delta = 0.069$

EIS and risk aversion
$\sigma = 0.667$ and $\gamma = 11$

β distribution

β	0.910000	0.999999
share of households	0.90	0.10

unemployment benefits $b_0 = 0.435$ of wage, pensions $b = 0.4$ times earnings

TFP and Labour Force Shocks

TFP growth process
$z' = \rho z + \varepsilon$ where $\varepsilon \sim \mathbf{N}\left(0, \sigma_\varepsilon^2\right)$ with $\rho = 0.909$ and $\sigma = 0.0109$.

Shock to Labour Force Participation and Investment
$N(u) = 2$ with $\pi_{11}^u = 0.98$ and $\pi_{22}^u = 0.9$.

Target 3.44 percent fall in labour force participation (2007Q3 - 2015Q3). LFP

2% probability of labour force shock when $u = 2$ (return with probability 0.5).

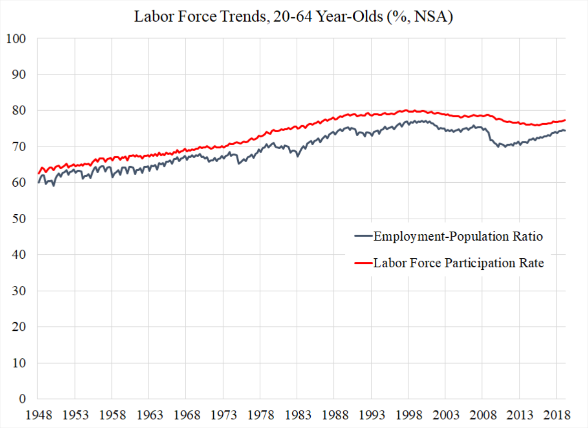
Investment TFP falls 20 percent

Capital Good production

$$K' = (1 - \delta) K + \left(a_0 + \frac{a_1}{\omega}\left(\frac{I}{K}\right)^{\omega}\right) K$$

$\omega = 0.65$, a_0 and a_1 set to imply no adjustment costs at unconditional mean of TFP growth (1.6346 per year)

Labour Force Participation

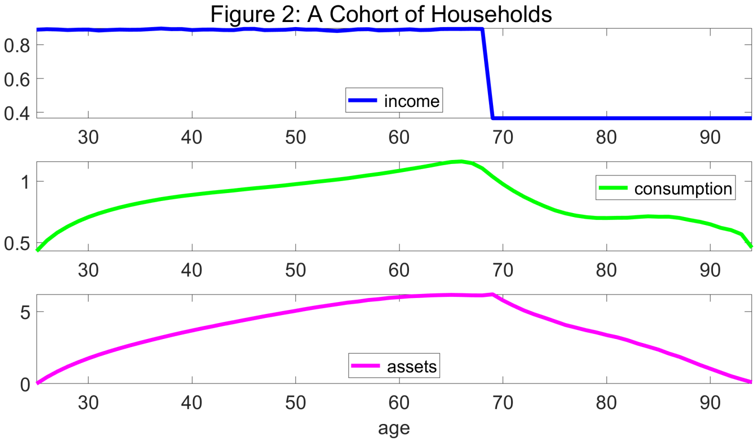


calibration

Nonemployment and uncertainty calibration

- Krusell, Mukoyama, Rogerson and Sahin (2017) data
  - monthly CPS aggregates, January 1978 - September 2012
  - adjusted for seasonality and classification error (Abowd and Zellner 1985)
- Adjust transition probabilities to account for retirement
- Transition matrices between employment, unemployment and non-employment vary with TFP growth and uncertainty
  - medium growth state matches pre-Great Recession averages
  - calibrate transitions for low and high TFP growth to match moments in data
- High uncertainty state adjusts transition to match severity of Great Recession

Cohort of Households



wealth distribution

Precautionary savings reduces inequality

comparison with CRRA

Wealth distribution with CRRA

	1	5	10	50	90	Gini	≤ 0
SCF	0.29	0.51	0.64	0.97	1.0	0.77	0.09
model	0.12	0.35	0.51	0.96	1.0	0.64	0.13

$\sigma = 2$, $r = 10.0\%$

Wealth distribution with CRRA

	1	5	10	50	90	Gini	≤ 0
SCF	0.29	0.51	0.64	0.97	1.0	0.77	0.09
model	0.22	0.75	0.94	1.01	1.01	0.92	0.46

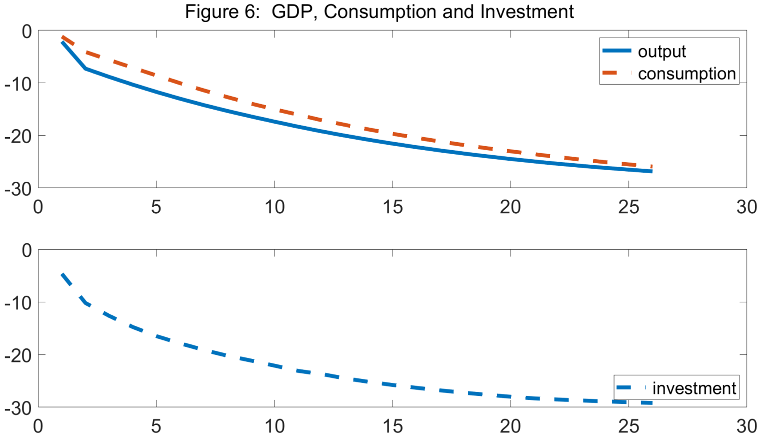
$\sigma = 0.667$, $r = 7.0\%$

- OLG economies imply inequality through life-cycle accumulation of wealth, Gini is 0.47 in Krusell Smith model with same earnings process.

▸ wealth distribution

GDP, Consumption and Investment

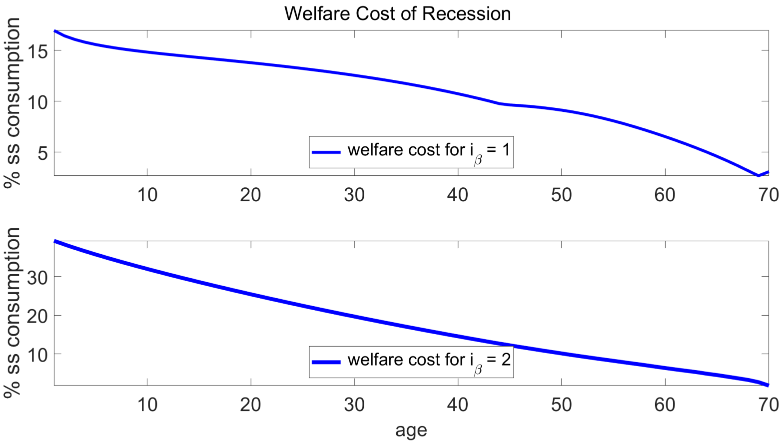
Monotone response in consumption.



stationary

Welfare Loss

large welfare costs of trend shocks



Largest welfare costs for *young* and patient households.

Comparison with CRRA economy

TFP growth shocks

Benchmark Model

Table 4: Business Cycle Moments

	Z	Y	C	I	N	K	w
std	1.86	2.53	1.97	2.91	1.68	2.67	1.71
correlation	0.82	1.00	0.94	0.98	0.74	0.62	0.75

Results from 1000 period simulations, HP-filtered with weight 100

CRRA Model with $\sigma = 2$

	Z	Y	C	I	N	K	w
std	1.86	2.53	1.94	3.21	1.68	2.69	1.72
correlation	0.82	1.00	0.94	0.98	0.74	0.61	0.75

Higher elasticity of intertemporal substitution in benchmark model ($\sigma = \frac{2}{3}$).

business cycles

Comparison with stationary economy

TFP without uncertainty

Benchmark Model

Table 4: Business Cycle Moments

	Z	Y	C	I	N	K	w
std	1.86	2.53	1.97	2.91	1.68	2.67	1.71
correlation	0.82	1.00	0.94	0.98	0.74	0.62	0.75

Results from 1000 period simulations, HP-filtered with weight 100

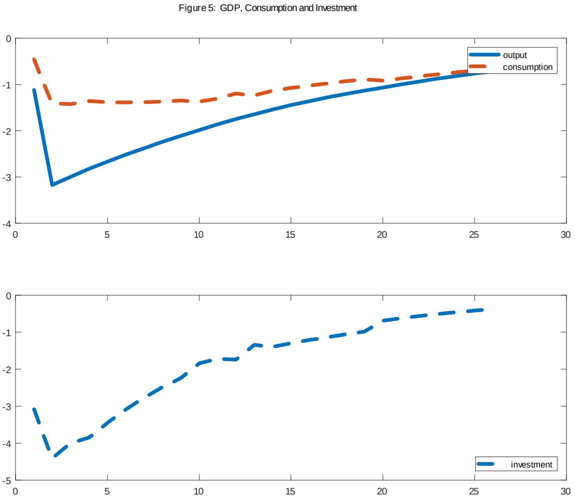
Stationary TFP

	Z	Y	C	I	N	K	w
std	1.86	1.98	1.09	3.56	1.68	1.81	1.15
correlation	0.72	1.00	0.85	0.98	0.82	0.00	0.53

Unemployment shocks are functions of last period TFP.

▸ business cycles

Stationary TFP

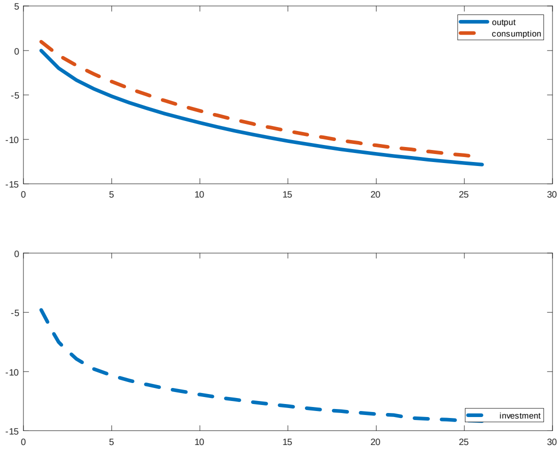


▸ GDP with TFP shock

Uncertainty with crra utility

large fall in investment

Figure 5: GDP, Consumption and Investment



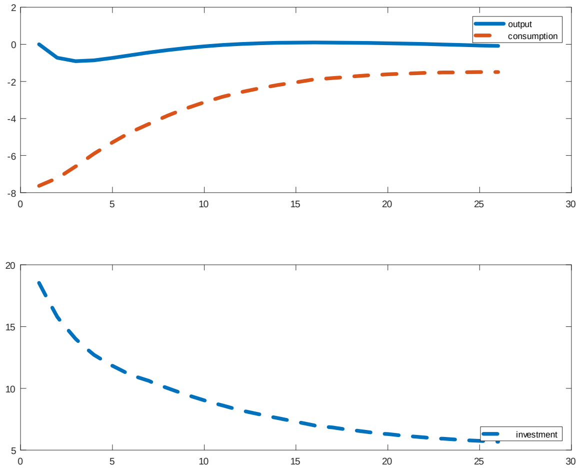
$\widehat{c} = -1.7\ (-3.4)$, $\widehat{i} = -8.9\ (-1.9)$, $\widehat{GDP} = -3.3\ (-3.0)$ ($t = 3$, LF falls $t = 2$)

▸ Uncertainty with Investment

Labour Force Participation Shock

Absence of investment productivity shock

Figure 5: GDP, Consumption and Investment



▸ Uncertainty with Investment

Next steps

Non-employment highly persistent in data

- large implication for precautionary savings

Approach similar to Krusell, Mukoyama, Rogerson and Sahin (2017)

- employed, unemployed or out of the labour force
- abstract from endogenous search choices
- transition matrices vary with aggregate tfp growth and uncertainty state

high uncertainty increases the persistence of out of labour force episodes